Wire Compensation of Long-Range Beam-Beam Effects

Béla Erdélyi

DOE Review 07-22-2003

Wire Compensation

- **Principle:** the strong beam is regarded as a current, so its effect on the weak beam could be alleviated by an appropriately placed current flowing in the opposite direction
- **Use** straight current carrying wires
- **Advantage:** simple, deals with all multipole orders at once
- **Difficulty** in applying it to the Tevatron: many parasitic encounters (72 per $\bar{p}$ bunch @ injection; 138 total locations), not enough wires
- **Question to be answered by simulations:** is it possible to compensate the long-range effects with only a few wires with constrained placement options?
- **What strategy** to be used to answer the question above, i.e. what do you optimize for?

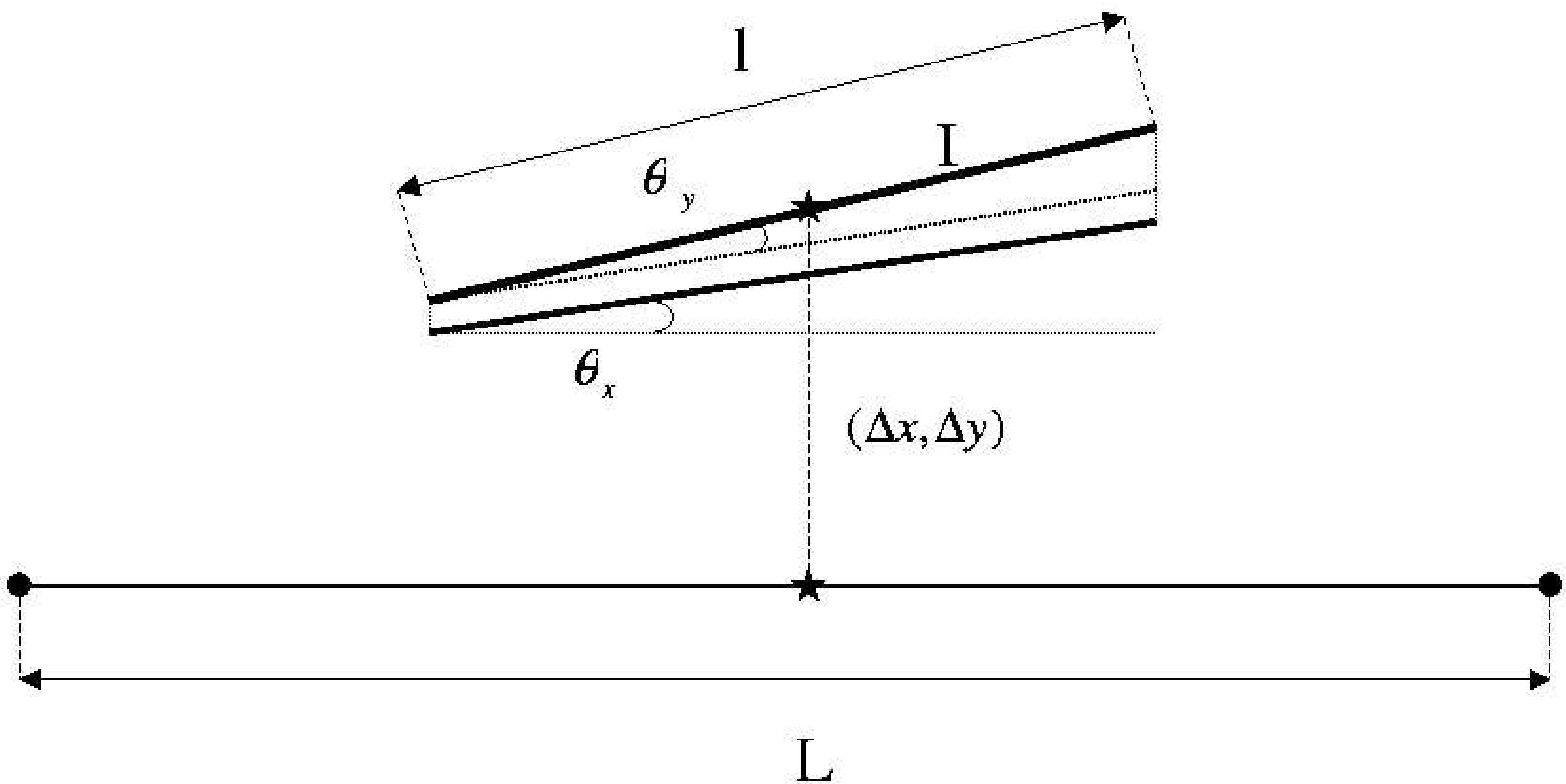
Transfer map of a wire

- **To track particles** with wire compensation accurately and allow optimization with ease, a more sophisticated implementation is needed
- **Requirements of the implementation:** fast, accurate, symplectic, includes finite length wires (i.e. fringe fields), allows misalignments
- **Analytical approach:** thin lens approximation (second order symplectic integration)
- **Numerical approach:** Differential Algebraic integration in COSY Infinity
- Very good agreement found

Details of the thin lens approximation

- **Zero length insertion** in a drift region, at an arbitrary distance from the closed orbit, any wire length, with any pitch and yaw
- Without loss of generality it is assumed that the tilts are with respect to the wire midpoints
- **The transfer map** of the whole element is composed of the following ones:
  - drift, backdrift
  - kick
  - tilt
  - shift
- The sequence is the following: backdrift, shift, inverse tilt, drift, kick, drift, tilt, backdrift

Outline of the wire model



Maps of the pieces

- Use canonical coordinates $\left(x, a=\frac{p_x}{p_0}, y, b=\frac{p_y}{p_0}\right)$
- **Drift** $(l>0)$ and **backdrift** $(l<0)$

$$\begin{cases} x_f = x_i + l\frac{a_i}{\sqrt{(1+\delta)^2-a_i^2-b_i^2}} \\ a_f = a_i \\ y_f = y_i + l\frac{b_i}{\sqrt{(1+\delta)^2-a_i^2-b_i^2}} \\ b_f = b_i \end{cases}$$

- **Shift**

$$\begin{cases} x_f = x_i - \Delta x \\ a_f = a_i \\ y_f = y_i - \Delta y \\ b_f = b_i \end{cases}$$

- **Tilt:** composed by two (noncommuting) tilts in the two transverse planes. In one plane it is

$$\begin{cases} x_f = x_i\left(\cos\theta_x - \sin\theta_x \tan\left(\alpha-\theta_x\right)\right) \\ a_f = \sqrt{\left(1+\delta\right)^2 - b_i^2}\,\sin\left(\alpha-\theta_x\right) \\ y_f = y_i - x_i\sin\theta_x\frac{b_i}{\sqrt{(1+\delta)^2-b_i^2}\cos(\alpha-\theta_x)} \\ b_f = b_i \end{cases}$$

The tilt in the other plane can be obtained by the replacements:

$$\begin{cases} x \leftrightarrow y \\ a \leftrightarrow b \end{cases}$$

Maps of the pieces (cont.)

- **Kick**

$$\begin{cases} x_f = x_i \\ a_f = a_i - \lambda \frac{x}{x^2+y^2} \left[\sqrt{\left[(L+l)^2 + x^2 + y^2 \right]} - \sqrt{\left[(L-l)^2 + x^2 + y^2 \right]} \right] \\ y_f = y_i \\ b_f = b_i - \lambda \frac{y}{x^2+y^2} \left[\sqrt{\left[(L+l)^2 + x^2 + y^2 \right]} - \sqrt{\left[(L-l)^2 + x^2 + y^2 \right]} \right] \end{cases}$$

  The kick is obtained from calculating analytically the field at any point in space using the Biot-Savart law, and the splitting of the Hamiltonian, which results in a system that can be integrated exactly in a coordinate system in which the z axis is parallel to the wire, resulting in the kick, which is exactly symplectic

- **The total map is:**

$$\mathcal{M}_{wire} = D_{-L_1} \circ S_{\Delta x, \Delta y} \circ T^{-1}_{\theta_x, \theta_y} \circ D_{L_2} \circ K \circ D_{L_3} \circ T_{\theta_x, \theta_y} \circ D_{-L_1}$$

  The total map, being a composition of symplectic maps, is also symplectic.

- **Implemented** in Sixtrack (vector - for tracking), COSY Infinity (map - for optimization) and in Mathematica

How to use the wire?

- **Optimization** in general is not well developed in tracking codes
- **Solution:** Differential Algebraic methods, i.e., compute a relatively high order truncation of the Taylor expansion of the one-turn map, with wire characteristics as parameters
- **The analysis of this map** by *normal form methods* painlessly gives all amplitude dependent tune shift, linear and nonlinear chromaticities, couplings, resonance driving terms, etc.

$$\mathcal{M}=\mathcal{A}^{-1}\circ\mathcal{N}\circ\mathcal{A}$$

$\mathcal{M}\to$truncated one-turn map

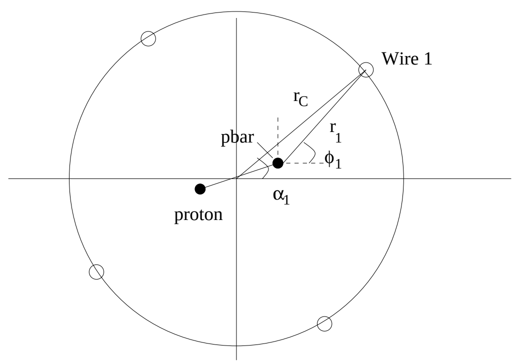
$\mathcal{N}\to$normal form,
containing the amplitude and
parameter dependence of the tunes

$\mathcal{A}\to$normalizing map,
containing the resonance terms

- Optimization in this setting is much more powerful
- COSY Infinity contains optimization commands at the language level
- **Possible optimization strategies:** minimize tune shifts and/or spreads, *resonance strengths*, a norm of the map, the distance of the map from identity in Hofer's metric
- Caveat: the complex error function and exponential, which enters the beam-beam kick is very slowly converging for amplitudes of practical interest. That is why COSY cannot be used for tracking and estimation of the DA. For tracking one needs to go back to a tracking code like Sixtrack

Wire Cages: Multiple Wires at a Single Location

- **Increase robustness and flexibility** by placing several wires around the surface of a cylindrical cage.

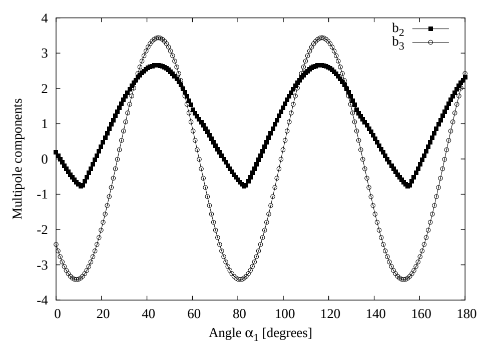


- Assume N_w wires placed at angles α_j w.r.t. the horizontal axis at distances r_j from the $\bar{\mathrm{p}}$ beam. Then the **multipoles** are (with $\langle r_w \rangle$ being the average distance)

$$b_n = -\sum_{j=1}^{N_w} \cos\left[(n+1)\,\alpha_j\right] \left(\frac{\langle r_w \rangle}{r_j}\right)^{n+1}$$

$$a_n = \sum_{j=1}^{N_w} \sin\left[(n+1)\,\alpha_j\right] \left(\frac{\langle r_w \rangle}{r_j}\right)^{n+1}$$

- The multipole content can be varied as a function of α_j and N_w



Work in Progress: Application to the Tevatron

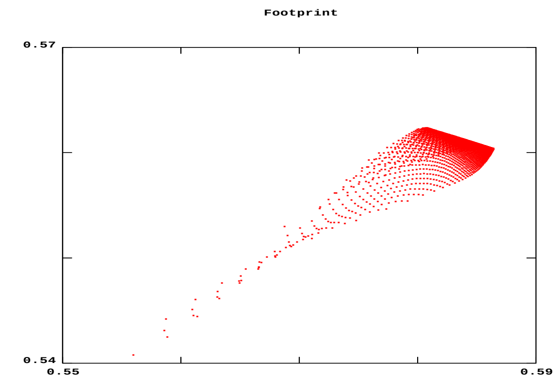
- **Number** of wires: 4
- **Longitudinal placement:** in available drifts where horizontal and vertical beta functions are not too different, and the proton and anti-proton beams are well separated
- **Transverse placement:** reasonable distance from both the beam pipe and $\bar{\text{p}}$ beams
- **Length:** fixed to 1 m
- **Current:** assuming round beams with design parameters ($N_b = 2.7 \cdot 10^{11}$), and kicks adding up linearly (gives upper bound for current)

$$I = 10^7 \left(\frac{r_p m_p c^2}{0.2998} \right) N_b \times \frac{72\,(\text{interactions})}{4\,(\text{wires})} = 232\ A$$

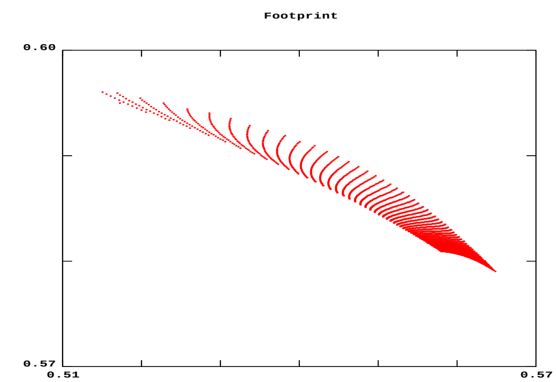
$\Longrightarrow$ Wires placed in sectors: A17, C0, E0, and F0

Normal Form: Tune Footprints from the Map

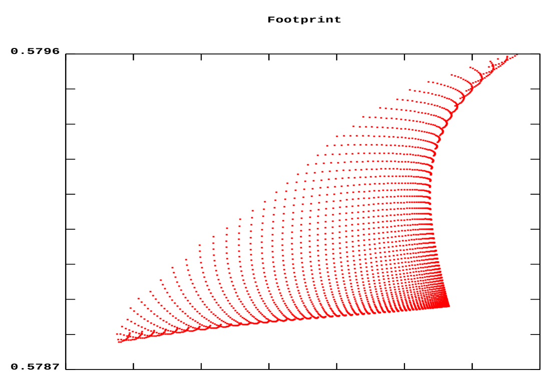
Injection, lattice only



Injection, lattice and wires

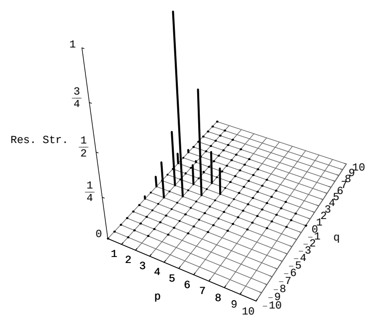


Collision, lattice only

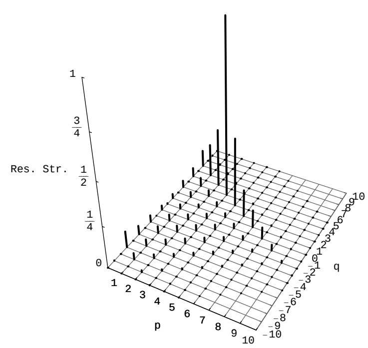


Normal Form: Resonance Strengths from the Map

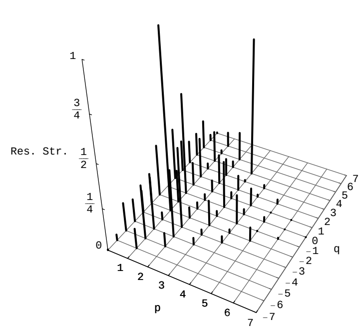
Injection, lattice only



Injection, lattice and beam-beam

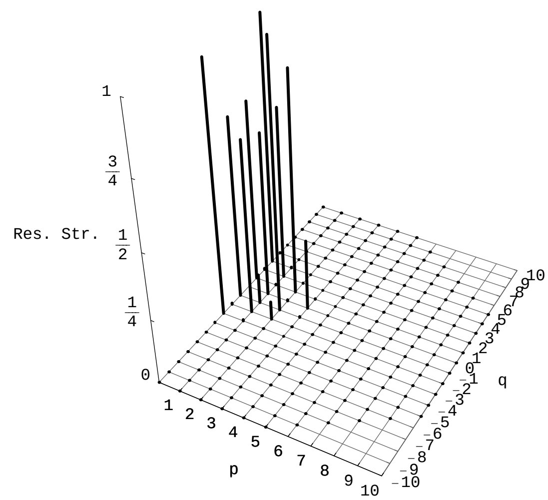


Injection, lattice and wires

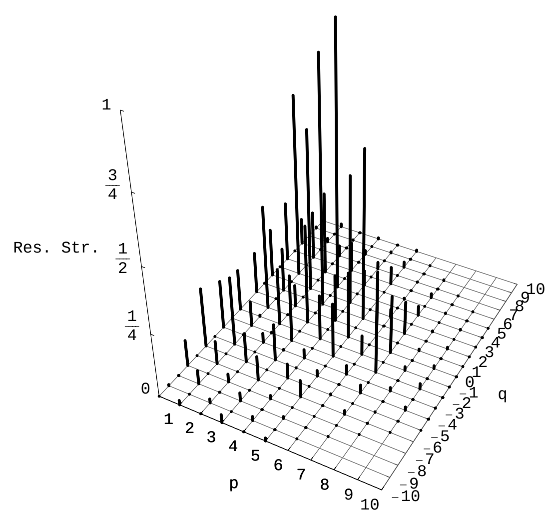


Normal Form: Resonance Strengths from the Map

Collision, lattice only



Collision, lattice and beam-beam



Summary and Outlook

- Developed (and continue developing) new tools to address the problem
- Identified optimization strategies
- New ideas to increase robustness and flexibility of correction
- Very preliminary results are encouraging, will start systematic simulations